ANNOTATION

for the degree dissertations on the topic " Solvability of non-local boundary value problems for quasi-hyperbolic equations of the fourth and sixth orders **" for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100 - "Mathematics" by Zhanat Bolatbayevna Sultangaziyeva**

Research topic: "Solvability of non-local boundary value problems for quasi-hyperbolic equations of the fourth and sixth orders "

The purpose of the study is to:

study of the solvability the local and nonlocal boundary value problems for the fourth and sixth orders quasi-hyperbolic equations; determination of the spectral parameter λ influence on the solvability of nonlocal boundary value problems for the sixth order quasi-hyperbolic equations.

Research objectives:

1. Formulation and to study of solvability of nonlocal boundary value problems with integral conditions on a time variable for fourth order quasihyperbolic equations.
2. Formulation and establishment of theorems solution's uniqueness and existence new boundary value problems for the fourth order quasihyperbolic equations;
3. Determination of the non-local boundary value problem's solvability for a hyperbolic equation with a fourth-order operator in a spatial variable.
4. Establishing theorems of uniqueness and existence of nonlocal boundary value problems for quasi-hyperbolic equations of the sixth order;
5. Determination of the spectral parameter λ influence on the solvability of nonlocal boundary value problems for the sixth order quasi-hyperbolic equations.

Research methods:

The works of the outstanding mathematician of the last century S.L. Sobolev enriched mathematical science with new ideas and research methods, laid the foundation for new theories and directions in mathematics. The concepts of generalized derivative and generalized solution introduced by him laid the foundations of the theory of generalized functions and were included in all modern textbooks on differential equations and equations of mathematical physics. Sobolev spaces, embedding theorems and integral representations of functions are an integral part of modern analysis. The methods of functionally invariant solutions are used in solving important applied problems of elasticity theory.

The differential operator equations we study relate to Sobolev type equations. The determination of the uniqueness criterion for solving initial boundary value problems for such differential operator equations is achieved in various ways. Various ways of proving uniqueness are known. Usually an effective means of proving uniqueness is the maximum principle and its various generalizations such as the Hopf and Zaremba-Giraud principles.

At the first stage of the study, the uniqueness of the solution of a non-local boundary value problem for a hyperbolic equation with a bi-Laplace operator is

proved. When solving this kind of non-local problems, the method of parameter continuation, the method of a priori estimation and the limit transition are usually used. For a hyperbolic equation, the continuation method by parameter is not applicable, since the smoothness of the right side of the equation will be lost. When proving the solvability theorems of nonlocal boundary value problems for quasihyperbolic equations of the fourth order, the method of parameter continuation, the method of a priori estimation, the limit transition and the Fourier method are used. As well as the Cauchy-Bunyakovsky and Young inequalities, the Galerkin method with the choice of a special basis.

The main provisions (proven scientific hypotheses and other conclusions that are new knowledge) submitted for defense

1. Statements are given and theorems of uniqueness and existence of solutions of nonlocal boundary value problems with integral conditions on a time variable for quasi-hyperbolic equations of the fourth order are established;

2. Statements are given and theorems of uniqueness and existence of solutions of new boundary value problems for quasi-hyperbolic equations of the fourth order are established;

3. The solvability of a nonlocal boundary value problem for a hyperbolic equation with a fourth-order operator in a spatial variable is established.

4. Theorems of uniqueness and existence of nonlocal boundary value problems for quasi-hyperbolic equations of the sixth order are established;

5. The effects of the spectral parameter λ on the solvability of nonlocal boundary value problems for quasi-hyperbolic equations of the sixth order are established.

Justification of the novelty and significance of the results obtained and their compliance with the directions of scientific development or government programs:

The substantiation of the novelty of the first results of scientific research consists in the fact that in this paper the issues of solvability of nonlocal problems with integral conditions in classes of regular solutions (i.e. solutions having all derivatives generalized by S.L.Sobolev included in the equation) are studied, the influence of the spectral parameter λ on uniqueness and non-uniqueness, existence and non-existence of the solution is investigated.

The second section consists of five subsections, which is devoted to the study of quasi-hyperbolic equations of the fourth order. The solvability of new boundary value problems is obtained for quasi-hyperbolic equations

$$u_{tttt} + \mathrm{A}u = f(x,t)$$

with an elliptic operator A acting on spatial variables, as well as for some of their generalizations.

For the studied problems, the existence and uniqueness theorems of regular (having all derivatives generalized by S.L. Sobolev included in the equation) solutions are proved, it is shown that when the conditions of the corresponding theorems are violated, the studied problems become incorrect. Examples of conditions for the numbers α_0, α_1 and $\alpha_2$are also given, under which both the conditions of the uniqueness theorem and the conditions of the existence theorem

will be fulfilled. The third section presents spectral problems for nonclassical differential equations of the sixth order. The effects of spectral parameters on the solvability of non-local boundary value problems are shown.

The results of the dissertation are theoretical in nature. It has developed a methodology for studying a number of boundary value problems for a quasi-hyperbolic equation of the fourth and sixth orders. They are investigated in certain functional spaces. In addition, the results obtained can serve as a definite contribution to the theory of partial differential equations of quasi-hyperbolic type. The practical value of the work is determined by the applied significance of a quasi-hyperbolic equation of the fourth order with non-local boundary conditions.

The reliability and validity of the conducted research is ensured by the constructiveness of the developed and used methods. Auxiliary statements of the problematic issues of each section are formulated in the form of lemmas and statements, and they are strictly proved, and general statements are presented in the form of theorems and their proofs are presented in a detailed presentation, as well as the reliability and novelty of the results of the work is confirmed by their publication in rating peer–reviewed publications included in the international scientometric databases Web of Science Core Collection and Scopus.

According to the results of the dissertation, reports were made at international conferences: the international scientific conference dedicated to the 90th anniversary of Sergei Konstantinovich Godunov "Mathematics in Applications" (Novosibirsk: Akademgorodok, 2019. - August 4-10), the international scientific conference dedicated to the 70th anniversary of Ph.D., Professor M.I. Ramazanov "Theoretical and applied issues of mathematics, mechanics and Computer science" (Karaganda: Karaganda State University named after Academician E.A.Buketov, 2019. – June 12-14), Voronezh Winter Mathematical School "Modern methods of the theory of functions of functions and related problems" (Voronezh: 2021. – January 28 – February 2), the traditional international April conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of the National Academy of Sciences of the Republic of Kazakhstan Academician T.Sh. Kalmenov (Almaty: IMMM, 2021). And also reports were made at the seminar under the guidance of professor Kozhanov A.I. (named after S.L.Sobolev SB RAS,), academician of NAS RK Otelbaev M.O., Professor Berdyshev A.S., Professor Koshanov B.D. (KazNPU named after Abaya, 2020), Professor B.E. Kanguzhin, Professor B.D. Koshanov (Al-Farabi Kazakh National University, 2023).

Description of the doctoral student's contribution to the preparation of each publication:

According to the results of the study, 4 papers were published, including 1 article with a percentile of 35 in an international peer-reviewed journal included in the scientometric database Scopus.

New boundary value problems for quasihyperbolic equations of the fourth order. Siberian Electronic Mathematical News. No. 16 (2019). p. 1410-1436. (Co-authors: Kozhanov A. I., Koshanov B. D., the share of the doctoral student is 70%). (WoS – Q4, Scopus – 35). http://semr.math.nsc.ru/v16/p1410-1436.pdf

In the article, the doctoral student investigated the solvability of new boundary value problems for quasi-hyperbolic equations with an elliptic operator A acting on spatial variables, as well as for some of their generalizations. For the problems under study, the existence and uniqueness theorems of regular (having all derivatives generalized by S.L. Sobolev included in the equation) solutions are proved.

The publications recommended by the Committee for quality assurance in the field of education and science and the Ministry of education and science of the Republic of Kazakhstan Published 3 articles:

1. Spectral problem for nonclassical differential equations of the sixth order. Bulletin of the E.A.Buketov Karaganda University, Mathematics Series No.1 (97) 2020, p.79-86. (Co-authors Kozhanov A.I., Koshanov B.D., Emir Kady oglu A.N., Smatova G.D., the share of the doctoral student 60%).(WoS – Q4,). https://mathematics-vestnik.ksu.kz/apart/srch/2020_mathematics_1_97_2020.pdf In the work, the applicant proved the existence and uniqueness theorems of boundary value problems for quasi-hyperbolic equations of the sixth order.

2. On the Schwarz problem for the Moisil-Teodorescu system in a spherical layer and in the interior of a torus. Bulletin of Al-Farabi Kazakh National University, Mathematics series. No. 2(114)2022. pp.35-42. (Co-authors: Koshanov B.D., Bayarystanov A.O., Dosmagulova K.A., Kuntuarova A.D. the share of a doctoral student is 60%) DOI: https://doi.org/10.26577/JMMCS.2022.v114.i2.04

3. Two theorems on estimates for solutions of one class of nonlinear equations in a finite-dimensional space. Bulletin of the Karaganda University named after E.A.Buketov, Mathematics series. No. 3(107)2022. pp.70-84. (Co-authors: Koshanov B.D., Kaharman N., Segizbaeva R.U., the share of the doctoral student is 70%) (WoS – Q4,). https://rep.ksu.kz//handle/data/13900